

Analysis of Solar Neutrino Data from SuperKamiokande I and II: Back to the Solar Neutrino Problem *

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(Received ; accepted)

Abstract

We are going back to the roots of the original solar neutrino problem: analysis of data from solar neutrino experiments. The application of standard deviation analysis (SDA) and diffusion entropy analysis (DEA) to the SuperKamiokande I and II data reveals that they represent a non-Gaussian signal. The Hurst exponent is different from the scaling exponent of the probability density function and both Hurst exponent and scaling exponent of the probability density function of the SuperKamiokande data deviate considerably from the value of 0.5 which indicates that the statistics of the underlying phenomenon is anomalous. To develop a road to the possible interpretation of this finding we utilize Mathai's pathway model and consider fractional reaction and fractional diffusion as possible explanations of the non-Gaussian content of the SuperKamiokande data.

Key words: methods: Sun: neutrinos — Sun: SuperKamiokande — methods: numerical

* Dedicated to the memory of Professor Dr. Masatoshi Kitamura, National Astronomical Observatory of Japan, who outstandingly supported annual United Nations workshop for the benefit of the worldwide development of astronomy

1. Introduction

This paper summarizes briefly a research programme, implemented through UN/ESA/NASA/JAXA workshops since 1991, comprised of five elements (i) standard deviation analysis and diffusion entropy analysis of solar neutrino data (Haubold et al. 2012a; Haubold et al. 2012b), (ii) Mathai's entropic pathway model (Mathai 2005; Mathai & Haubold 2007), (iii) fractional reaction and extended thermonuclear functions (Haubold & Mathai 2000; Kumar & Haubold 2010), (iv) fractional reaction and diffusion (Saxena et al. 2004; Saxena et al. 2010), and (v) fractional reaction-diffusion (Saxena et al. 2007a, Saxena et al. 2007b, Saxena et al. 2007c; Haubold & Mathai 2011). Boltzmann translated Clausius' second law of thermodynamics "The entropy of the Universe tends to a maximum" into a crucial quantity that links equilibrium and non-equilibrium (time dependent) properties of physical systems and related entropy to probability, $S = k \log W$, which later Einstein called Boltzmann's principle (Brush 1976). Based on this principle of physics, Planck found the correct formula for black-body radiation that led him to the discovery of the elementary quantum of action that initiated the development of quantum theory. Extremizing the Boltzmann entropic functional under appropriate constraints produces the exponential functional form of the distribution for the respective physical quantity. Today a question under intense discussion in statistical mechanics is on how to generalize Boltzmann's entropic functional, if extremized under appropriate constraints, to accommodate power law distribution functions observed so frequently in nature. One of such generalizations is Tsallis statistics (Tsallis 2009) that contains Boltzmann statistics as a special case. Tsallis statistics is characterized by q -distributions which seem to occur in many situations of scientific interest and have significant consequences for the understanding of natural phenomena. One of such phenomena concerns the neutrino flux emanating from the gravitationally stabilized solar fusion reactor (Degl'Innocenti et al. 1998; Wolff 2009). R. Davis Jr. established the solar neutrino problem which was resolved by the discovery of neutrino oscillations (Smirnov 2003; Pulido et al. 2010). A remaining question to date is still the quest for more information hidden in solar neutrino records of numerous past and currently operating solar neutrino experiments (Oser 2012). Greatly stimulated by the question, raised long time ago, by R.H. Dicke "Is there a chronometer hidden deep in the Sun?" (Dicke 1978; Perry 1990), Mathai's research programme on the analysis of the neutrino emission of the gravitationally stabilized solar fusion reactor, focused on non-locality (long-range correlations), non-Markovian effects (memory), non-Gaussian processes (Lévy), and non-Fickian diffusion (scaling), possibly evident in the solar neutrino records, taking also into account results of helio-seismology and helio-neutrinospectroscopy (Goupil 2011). The original research programme, devised by Mathai, is contained in three research

and astrophysics over a period of time of a quarter of a century (W. Wamsteker et al. 2004, Developing Basic Space Science World-Wide: A Decade of UN/ESA Workshops, (Dordrecht Boston London: Kluwer Academic Publishers); B.J. Thompson et al. 2009, Putting the "I" in IHY: The United Nations Report for the International Heliophysical Year 2007, (Wien New York: Springer); <http://www.iaucomm46.org/content/united-nations-basic-space-science-initiative-unbssi-1991-2012>)

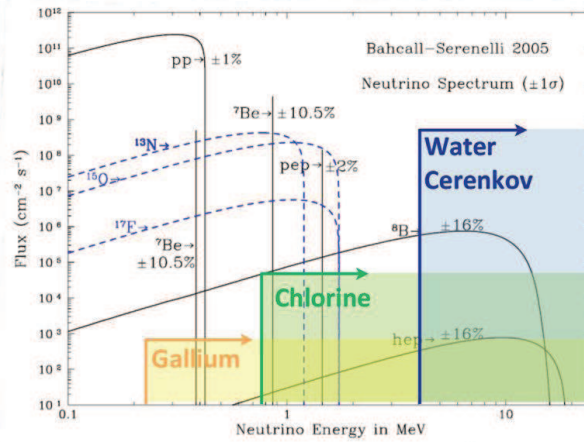


Fig. 1. The solar neutrino spectrum for the pp-chain and the CNO-cycle and parts of the spectrum that are being detectable by the experiments based on gallium, chlorine, and Cherenkov radiation (Haxton et al. 2012).

monographs (Mathai & Rathie 1975; Mathai & Pederzoli 1977; Mathai & Saxena 1978) and the results of this research programme were summarized recently in Mathai et al. 2010.

2. Solar Neutrino Data

Over the past 40 years, radio-chemical and real-time solar neutrino experiments have proven to be sensitive tools to test both astrophysical and elementary particle physics models and principles. Solar neutrino detectors (radio-chemical: Homestake, GALLEX + GNO, SAGE, real-time: SuperKamiokande, SNO, Borexino) (Oser 2012; Haxton et al. 2012) have demonstrated that the Sun is powered by thermonuclear fusion reactions. Two distinct processes, the pp-chain and the sub-dominant CNO-cycle, are producing solar neutrinos with different energy spectra and fluxes (see Figure 1). To date only fluxes from the pp-chain have been measured: ${}^7\text{Be}$, ${}^8\text{B}$, and, indirectly, pp . Experiments with solar neutrinos and reactor anti-neutrinos (KamLAND; see Haxton et al. 2012) have confirmed that solar neutrinos undergo flavor oscillations (Mikheyev-Smirnov-Wolfenstein (MSW) model; see Pulido et al. 2010). Results from solar neutrino experiments are consistent with the Mikheyev-Smirnov-Wolfenstein Large Mixing Angle (MSW-LMA) model, which predicts a transition from vacuum-dominated to matter-enhanced oscillations, resulting in an energy dependent electron neutrino survival probability. Non-standard neutrino interaction models derived such neutrino survival probability curves that deviate significantly from MSW-LMA, particularly in the 1-4 MeV transition region. The mono-energetic 1.44 MeV pep neutrinos, which belong to the pp-chain and whose Standard Solar Model (SSM) predicted flux has one of the smallest uncertainties due to the solar luminosity constraint, are an ideal probe to test these competing non-standard neutrino interaction models in the future (Ludhova et al. 2012).

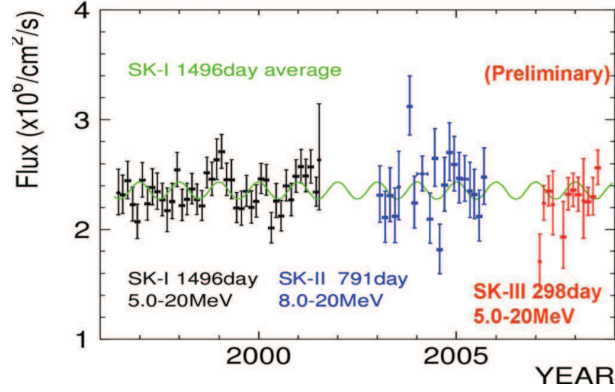


Fig. 2. The variation of the solar neutrino flux over time as shown in the SuperKamiokande I, II, and III experiments (Yoo 2003; Cravens et al. 2008; Abe et al. 2011).

3. Standard Deviation Analysis and Diffusion Entropy Analysis

For all radio-chemical and real-time solar neutrino experiments, periodic variation in the detected solar neutrino fluxes have been reported, based mainly on Fourier and wavelet analysis methods (standard deviation analysis) (Davis et al. 1987; Sakurai et al. 2008; Vecchio & Carbone 2009; Vecchio et al. 2010). Other attempts to analyze the same data sets, particularly undertaken by the experimental collaborations of real-time solar neutrino experiments themselves, have failed to find evidence for such variations of the solar neutrino flux over time. Periodicities in the solar neutrino fluxes, if confirmed, could provide evidence for new solar, nuclear, or neutrino physics beyond the commonly accepted physics of vacuum-dominated and matter-enhanced oscillations of massive neutrinos (MSW model) that is, after 40 years of solar neutrino experiment and theory, considered to be the ultimate solution to the solar neutrino problem. Specifically, subsequent to the analysis made by the SuperKamiokande collaboration (Yoo 2003; Cravens et al. 2008; Abe et al. 2011), the SNO experiment collaboration has painstakingly searched for evidence of time variability at periods ranging from 10 *years* down to 10 *minutes*. SNO has found no indications for any time variability of the 8B flux at any timescale, including in the frequency window in which g -mode oscillations of the solar core might be expected to occur (Aharmim et al. 2010). Despite large efforts to utilize helioseismology and helio-neutrinospectroscopy, at present time there is no conclusive evidence in terms of physics for time variability of the solar neutrino fluxes from any solar neutrino experiment (Pulido et al. 2010; Goupil 2011). If such a variability over time would be discovered, for example in the Borexino experiment, a mechanism for a chronometer for solar variability could be proposed based on relations between properties of thermonuclear fusion and g -modes. All above findings encouraged the conclusion that Fourier and wavelet analysis, which are based upon the analysis of the variance of the respective time series (standard deviation analysis: SDA) (Haubold & Gerth 1990; Haubold & Mathai 1998), should be complemented by the utilization of diffusion entropy analysis (DEA), which measures the scaling of the probability density function (pdf) of the diffusion process gener-

ated by the time series thought of as the physical source of fluctuations (Scafetta & Latora 2002; Scafetta 2010). For this analysis, we have used the publicly available data of SuperKamiokande-I and SuperKamiokande-II (see Figure 2). Such an analysis does not reveal periodic variations of the solar neutrino fluxes but shows how the pdf scaling exponent departs in the non-Gaussian case from the Hurst exponent. Figures 3 to 6 show the Hurst exponents (SDA) and scaling exponents (DEA) for the SuperKamiokande I and II data. SuperKamiokande is sensitive mostly to neutrinos from the 8B branch of the pp nuclear fusion chain in solar burning. Above approximately 4 MeV the detector can pick-out the scattering of solar neutrinos off atomic electrons which produces Cherenkov radiation in the detector. The 8B and rarer hep neutrinos have a spectrum which ends near 20 MeV (see Figure 1).

Assuming that the solar neutrino signal is governed by a probability density function with scaling given by the asymptotic time evolution of a pdf of x , obeying the property

$$p(x, t) = \frac{1}{t^\delta} F\left(\frac{x}{t^\delta}\right), \quad (1)$$

where δ denotes the scaling exponent of the pdf. In the variance based methods, scaling is studied by direct evaluation of the time behavior of the variance of the diffusion process. If the variance scales, one would have

$$\sigma_x^2(t) \sim t^{2H}, \quad (2)$$

where H is the Hurst exponent. To evaluate the Shannon entropy of the diffusion process at time t , Scafetta et al. (2002; Scafetta 2010) defined $S(t)$ as

$$S(t) = - \int_{-\infty}^{+\infty} dx p(x, t) \ln p(x, t) \quad (3)$$

and with the previous $p(x, t)$ one has

$$S(t) = A + \delta \ln(t), \quad A = - \int_{-\infty}^{+\infty} dy F(y) \ln F(y) \quad (4)$$

The scaling exponent δ is the slope of the entropy against the logarithmic time scale. The slope is visible in Figures 4 and 6 for the SuperKamiokande data measured for 8B and hep . The Hurst exponents (SDA) are $H = 0.66$ and $H = 0.36$ for 8B and hep , respectively, shown in Figures 3 and 5. The pdf scaling exponents (DEA) are $\delta = 0.88$ and $\delta = 0.80$ for 8B and hep , respectively, as shown in Figures 4 and 6. The values for both SDA and DEA indicate a deviation from Gaussian behavior which would require that $H = \delta = 0.5$. A preliminary analysis, for SuperKamiokande I data exclusively, was undertaken recently by Haubold et al. (2012a). A test computation for the application of SDA and DEA to data that are known to exhibit non-Gaussian behavior have been published recently by Haubold et al. (2012b). In this test, SDA and DEA, applied to the magnetic field strength fluctuations recorded by the Voyager-I spacecraft in the heliosphere clearly revealed the scaling behavior of such fluctuations as previously already discovered by non-extensive statistical mechanics considerations that lead to the determination of the non-extensivity q -triplet (Tsallis 2009).

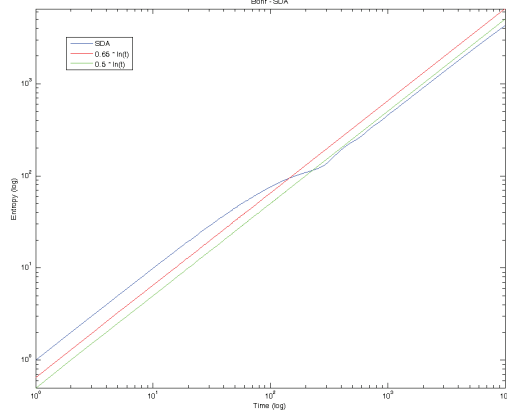


Fig. 3. The Standard Deviation Analysis (SDA) of the 8B solar neutrino data from the SuperKamiokande I and II experiment.

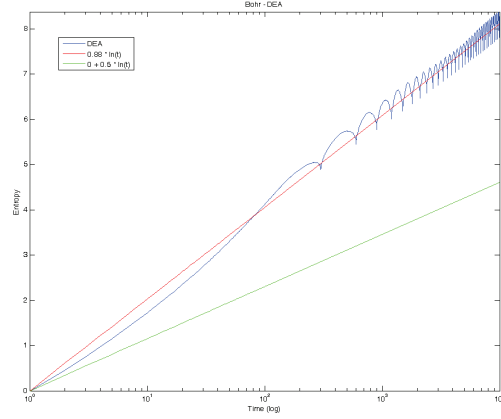


Fig. 4. The Diffusion Entropy Analysis (DEA) of the 8B solar neutrino data from the SuperKamiokande I and II experiment.

4. Mathai's Entropic Pathway Model

From a general point of view of fitting experimental data to mathematical functions, a model, which moves from the generalized type-1 beta family to the type-2 beta family to the generalized gamma family to the generalized Mittag-Leffler family and eventually to the Lévy distributions, has been developed by Mathai Mathai 2005. All these different parametric families of functions are connected through Mathai's pathway parameter $\alpha > 1$. To generalize Shannon's entropy to an entropic pathway, Mathai introduced the generalized entropy of order α that is also associated with Shannon (Boltzmann-Gibbs), Renyi, Tsallis, and Harvrda-Charvat entropies (Mathai & Haubold 2007; Mathai et al. 2010). Applying the maximum entropy principle with normalization and energy constraints to Mathai's entropic functional, the corresponding parametric families of distributions of generalized

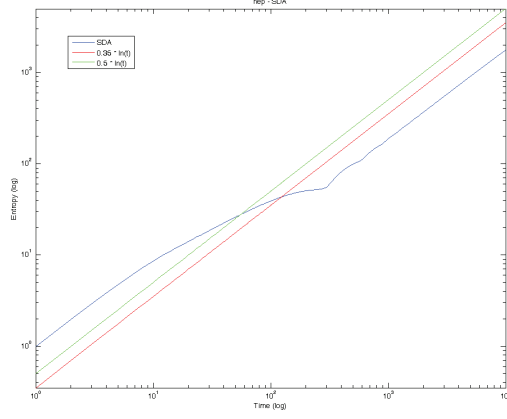


Fig. 5. The Standard Deviation Analysis (SDA) of the *hep* solar neutrino data from the SuperKamiokande I and II experiment.

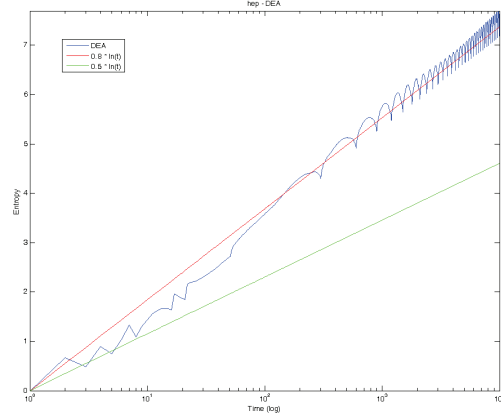


Fig. 6. The Diffusion Entropy Analysis (DEA) of the *hep* solar neutrino data from the SuperKamiokande I and II experiment.

type-1 beta, type-2 beta, generalized gamma, generalized Mittag-Leffler, and Lévy are obtained in the following form

$$M_2(f) = \frac{\int_{-\infty}^{+\infty} dx [f(x)]^{2-\alpha} - 1}{\alpha - 1} \quad \alpha \neq 1, \alpha < 2, \quad (5)$$

$$f(x) = c_1 [1 - \beta(1 - \alpha)x^\delta]^{1/(1-\alpha)} \quad (6)$$

with $\alpha < 1$ for type-1 beta, $\alpha > 1$ for type-2 beta, $\alpha \rightarrow 1$ for gamma, and $\delta = 1$ for Tsallis statistics. In principle, any entropic functional in Mathai's pathway can be tested through the above Diffusion Entropy Analysis against experimental data. The deviation of the statistical properties of the SuperKamiokande data analyzed above from Gaussian should be captured by equation (6).

5. Fractional Reaction and Extended Thermonuclear Functions

Solar nuclear reactions, producing neutrinos, occur preferably between nuclei in the high-energy tail of the energy distribution and are sensitive to deviations from the standard equilibrium thermal energy distribution (Maxwell-Boltzmann distribution) (Critchfield 1972; Degl'Innocenti et al. 1998; Kumar & Haubold 2010). Reaction and relaxation processes in thermonuclear plasmas are governed by ordinary differential equations of the type

$$\frac{dN(t)}{dt} = c N(t) \quad (7)$$

for exponential behavior. The quantity c is a thermonuclear function which is governed by the average of the Gamow penetration factor over the Maxwell-Boltzmannian velocity distribution of reacting species and has been extended to incorporate more general distributions than the normal distribution (Mathai et al. 2010). The coefficient c itself can be considered to be a statistical quantity subject to accommodating a distribution of its own (Mathai & Haubold 2007). To address non-exponential properties of a reaction or relaxation process, the first-order time derivative can be replaced formally by a derivative of fractional order in the following way (Mathai et al. 2010)

$$N(t) = N_0 - c^\nu {}_0D_t^{-\nu} N(t), \quad (8)$$

where ${}_0D_t^{-\nu}$ denotes a Riemann-Liouville fractional integral operator, and the solution can be represented in terms of Mittag-Leffler functions E_ν by

$$N(t) = N_0 E_\nu(-c^\nu t^\nu). \quad (9)$$

Considering c to be a random variable itself, $N(t)$ is to be taken as $N(t | c)$ and can be written as

$$N(t | c) = N_0 t^{\mu-1} E_{\nu,\mu}^{\gamma+1}(-c^\nu t^\nu), \quad \mu > 0, \gamma > 0, \nu > 0, \quad (10)$$

which represents a generalized Mittag-Leffler function, and is a random variable having a gamma type density

$$g(c) = \frac{\omega^\mu}{\Gamma(\mu)} c^{\mu-1} e^{-\omega c} \quad \omega > 0, 0 < c < \infty, \mu > 0, \quad (11)$$

with μ/ω is the mean value of c . The integration of $N(t | c)$ over $g(c)$ gives the unconditional density, as

$$N(t) = \frac{N_0}{\Gamma(\mu)} t^{\mu-1} [1 + b(\alpha - 1)t^\nu]^{-1/(\alpha-1)}, \quad (12)$$

with $\gamma + 1 = 1/(\alpha - 1)$, $\alpha > 1 \rightarrow \gamma = (\alpha - 2)/(\alpha - 1)$ and $\omega^{-\nu} = b(\alpha - 1)$, $b > 0$, which corresponds to Tsallis statistics for $\mu = 1, \nu = 1, b = 1$, and $\alpha = q > 1$, physically meaning that the common exponential behavior is replaced by a power-law behavior, including Lévy statistics. Both the translation of the standard reaction equation (7) to a fractional reaction equation (8) and the probabilistic interpretation of such equations lead to deviations from the exponential behavior to power law behavior expressed in terms of Mittag-Leffler functions (9) or, as can be shown for

equation (12), to power law behavior in terms of H-functions (Mathai et al. 2010). H-functions are representable in terms of Mellin-Barnes integrals of the product of gamma functions and are therefore suited to represent statistics of products and quotients of independent random variables thus providing a very useful tool in presenting a new perspective on the statistics of random variables (Cottone et al. 2010).

6. Fractional Diffusion and the Joint Action of Reaction and Diffusion

In recent time, an analytic approach to non-conventional reaction and diffusive transport by taking into account fractional space and time derivatives has been developed (Del-Castillo 2008). The probability density function for the above SuperKamiokande data is non-Gaussian and exhibits stretched power-law tails as can be shown by further exploring equations (6), (9), and (12). In order to model these analytic findings, a transport model for the pdf, based on fractional diffusion that includes both non-local and non-Gaussian features was proposed (Mathai et al. 2010). Reaction and diffusion in the solar thermonuclear fusion plasma are non-linear phenomena that may be subject to non-Fickian transport (non-locality), non-Markovian effects (memory), and non-Gaussian scaling (Lévy). Fractional diffusion operators are integro-differential operators that incorporate the former three phenomena in a natural way and may be, in this regard, constitute spatio-temporal elements of fundamental theory of physics. This issue is currently under intense research. Continues time random walk (CTRW) balance equations (master equations) with temporal memory, generation/destruction terms, and spatio transport/relaxation elements yield non-linear fractional reaction-diffusion equations whose solutions are a focus of current research and only very special cases have been dealt with so far. Equally difficult to reveal is the interplay between fractional reaction and fractional diffusion in such non-linear equations. This difficulty is amplified by the fact that various definitions of fractional operators exist (Riemann-Liouville, Caputo, Weyl, Gruenwald-Letnikov, Riesz-Feller, ...). At this point of time there is no general understanding under which specific mathematical and physical conditions a probabilistic interpretation can be given to unified fractional reaction-diffusion equations. And this difficulty is even further amplified by the observation that the replacement of integer order with fractional order time derivatives changes the fundamental concept of time and violates the principle that time evolution (change) is time translation and that fractional order space derivatives are bridging the respective differential equation between the case of diffusion equation and wave equation.

7. Conclusion

The use of solar neutrino detection records, by analyzing the average neutrino flux of the experiments, have lead to the discovery of new elementary particle physics, the MSW effect, and thus resolved the solar neutrino problem. This confirmed that the Standard Solar Model is implementing physical principles correctly. The quest for the variation of the solar neutrino flux over time remains

an open question. Additionally, the utilization of standard deviation analysis (scaling of the variance) and diffusion entropy analysis (scaling of the pdf) lead to the discovery of an unknown phenomenon related to non-equilibrium signature in the gravitationally stabilized solar fusion reactor as explored by looking at Mathai's pathway model and taking into account fractional reaction and fractional diffusion and possibly a combination of both of them.

The authors acknowledge the cooperation of Dr. Alexander Haubold of the Department of Computer Science, Columbia University, New York. The authors would like to thank the Department of Science and Technology, Government of India, New Delhi, for the financial assistance under project SR/S4/MS:287/05.

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